# TRANSACTIONS 

OF THE

## SOUTH AFRICAN PHILOSOPHICAL SOCIETY.

## NOTE ON THE THREE-POINT, OR POTHENOT'S, PROBLEM.

By H. G. Fourcade.

(Read January 27, 1897.)
The ordinary methods of computation of the position of a point, given the angles subtended by three other points of known positions are chiefly :-

First method.


B 1. Compute the length and "angle of direction" of $a$ from the co-ordinates of C and A .
2. Compute the length and angle of direction of $\beta$ from C and B .
3. Put PAC $=x, \mathrm{PBC}=y$. Then $\tan \frac{1}{2}(x-y)=\tan \left(z-45^{\circ}\right) \tan \frac{1}{2}(x+y)$ Where

$$
\tan z=\frac{a \sin \beta}{b \sin a}
$$

and

$$
{ }_{51}{ }_{51}^{\frac{1}{2}(x+y)=180^{\circ}-\frac{1}{2}(\alpha+\beta+\mathrm{C}) .}
$$

4. Compute the co-ordinates of P either from triangle PCA or triangle PBC .

## Second method.



1. Compute the length and angle of direction of C from the co-ordinates of $A$ and $B$.
2. Compute the co-ordinates of O from the triangle OAB in which $\mathrm{OAB}=\beta$ and $\mathrm{OBA}=\alpha$.
3. Compute the angle of direction of OC from the co-ordinates of O and C .
4. Compute the co-ordinates of P either from triangle POA or PBO.

Both these methods are avoided by many surveyors on account of their length. A shorter method will now be given, with a numerical example showing the arrangement of the computation.

Taking the middle point C for origin, put $x^{\prime} y^{\prime}$ and $x^{\prime \prime} y^{\prime \prime}$ for the co-ordinates of A and B . The equations to the circles (1) through A and C and containing the angle $a(2)$ through C and B and containing the angle $\beta$ are

$$
\begin{aligned}
& \tan a\left\{y\left(y-y^{\prime}\right)+x\left(x-x^{\prime}\right) \succ-x y^{\prime}+y x^{\prime}=0\right. \\
& \tan \beta\left\{y\left(y-y^{\prime \prime}\right)+x\left(x-x^{\prime \prime}\right)\right\}-y x^{\prime \prime}+x y^{\prime \prime}=0
\end{aligned}
$$

reducible to

$$
\begin{aligned}
& y^{2}+x^{2}+\mathrm{A} y-\mathrm{B} x=\mathrm{O} \\
& y^{2}+x^{2}-\mathrm{C} y+\mathrm{D} x=\mathrm{O}
\end{aligned}
$$

Where

$$
\begin{array}{ll}
\mathrm{A}=x^{\prime} \cot a-y^{\prime} & \mathrm{B}=y^{\prime} \cot a+x^{\prime} \\
\mathrm{C}=x^{\prime \prime} \cot \beta+y^{\prime \prime} & \mathrm{D}=y^{\prime \prime} \cot \beta-x^{\prime \prime}
\end{array}
$$

Then

$$
\begin{gathered}
\frac{y}{x}=\frac{\mathrm{B}+\mathrm{D}}{\mathrm{~A}+\mathrm{C}}=m \\
m^{2} x+x=\mathrm{B}-m \mathrm{~A} \\
x=\frac{\mathrm{B}-m \mathrm{~A}}{m^{2}+1} \quad y=m x
\end{gathered}
$$

| Example. |  |  |  |
| :---: | :---: | :---: | ---: |
| $\mathrm{A}-1811 \cdot 59$ | $-1018 \cdot 55$ | $y^{\prime}=-1376 \cdot 55$ | $x^{\prime}=+406 \cdot 90$ |
| $\mathrm{~B}+\quad 6 \cdot 81$ | $-930 \cdot 26$ | $y^{\prime \prime}=+441 \cdot 85$ | $x^{\prime \prime}=+495 \cdot 19$ |
| $\mathrm{C}-435 \cdot 04$ | $-1425 \cdot 45$ | $0 \cdot 0$ | $0 \cdot 00$ |
| $a=64 \cdot 7 \cdot 40$ |  |  |  |
| $\beta=20 \cdot 33 \cdot 20$ |  |  |  |
| $+9 \cdot 685719$ | $+9 \cdot 685719$ | $+0 \cdot 425980$ | $+0 \cdot 425980$ |
| $+2 \cdot 609488$ | $-3 \cdot 138792$ | $+2 \cdot 694772$ | $+2 \cdot 645275$ |
| $+2 \cdot 295207$ | $-2 \cdot 824511$ | $+3 \cdot 120752$ | $+3 \cdot 071255$ |
| $+197 \cdot 34$ | $-667 \cdot 59$ | $+1320 \cdot 54$ | $+1178 \cdot 30$ |
| $-y^{\prime}+1376 \cdot 55$ | $+x^{\prime}+406 \cdot 90$ | $+y^{\prime \prime}+441 \cdot 85$ | $-x^{\prime \prime}-495 \cdot 19$ |
| $\mathrm{~A}+1573 \cdot 89$ | $\mathrm{~B}-260 \cdot 69$ | $\mathrm{C}+1762 \cdot 39$ | $\mathrm{D}+683 \cdot 11$ |
| $\mathrm{~A}+3 \cdot 196974$ | $-2 \cdot 662730$ | $\mathrm{~A}+1573 \cdot 89$ | $\mathrm{~B}-260 \cdot 69$ |
| $m 9 \cdot 102482$ | $0 \cdot 006906$ | $+3336 \cdot 28$ | $+422 \cdot 42$ |
| $+2 \cdot 299456$ | $x-2 \cdot 655824$ | $2 \cdot 625744$ | $m 9 \cdot 102482$ |
| $-m \mathrm{~A}-199 \cdot 28$ | 9102482 | $3 \cdot 523262$ | $m^{2} 8 \cdot 204964$ |
| $+\mathrm{B}-260 \cdot 69$ | $y-1 \cdot 758306$ | $\frac{m^{2}+1}{}$ | $=1 \cdot 01603$ |
| $-459 \cdot 97$ |  | $y-57 \cdot 32$ | $x-452 \cdot 71$ |
|  |  | $-435 \cdot 04$ | $-1425 \cdot 45$ |
|  | Co-ordinates of $\mathrm{P}:$ | $-492 \cdot 36$ | $-1878 \cdot 16$ |

A check is afforded by the computation of the angles of direction P A and P B. P C is given by

$$
\tan -\mathrm{r} m .
$$

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