TRANSACTIONS

OF THE

SOUTH AFRICAN PHILOSOPHICAL SOCIETY.

NOTE ON THE THREE-POINT, OR POTHENOT'S, PROBLEM.

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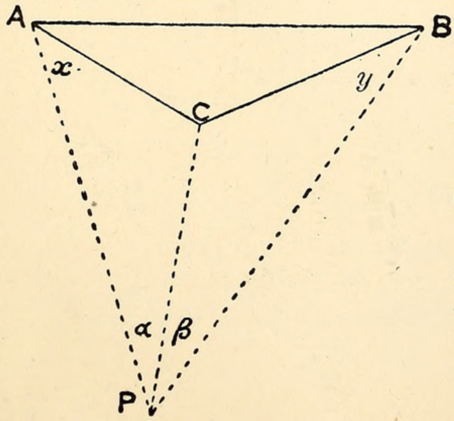
(Read January 27, 1897.)

The ordinary methods of computation of the position of a point, given the angles subtended by three other points of known positions are chiefly :—

First method.



1. Compute the length and "angle of direction" of α from the co-ordinates of C and A.

2. Compute the length and angle of direction of β from C and B.

3. Put $PAC = x$, $PBC = y$. Then $\tan \frac{1}{2}(x-y) = \tan (z-45^\circ) \tan \frac{1}{2}(x+y)$ Where

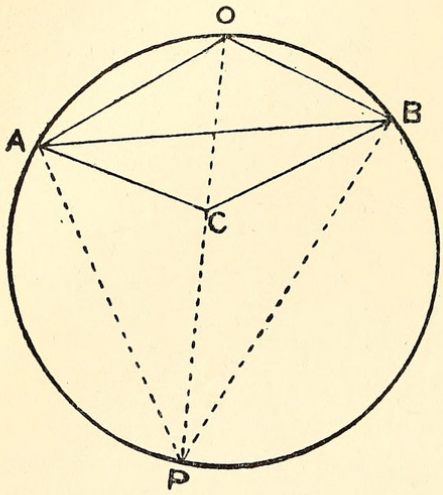
$$\tan z = \frac{a \sin \beta}{b \sin \alpha}$$

and

$$\frac{1}{2}(x+y) = 180^\circ - \frac{1}{2}(\alpha + \beta + C).$$

4. Compute the co-ordinates of P either from triangle P C A or triangle P B C.

Second method.



1. Compute the length and angle of direction of C from the co-ordinates of A and B.

2. Compute the co-ordinates of O from the triangle O A B in which O A B $=\beta$ and O B A $=\alpha$.

3. Compute the angle of direction of O C from the co-ordinates of O and C.

4. Compute the co-ordinates of P either from triangle P O A or P B O.

Both these methods are avoided by many surveyors on account of their length. A shorter method will now be given, with a numerical example showing the arrangement of the computation.

Taking the middle point C for origin, put x' y' and x'' y'' for the co-ordinates of A and B. The equations to the circles (1) through A and C and containing the angle α (2) through C and B and containing the angle β are

$$\tan\alpha\left\{y\,(y-y')+x\,(x-x')\right\}-xy'+yx'=0$$
$$\tan\beta\left\{y\,(y-y'')+x\,(x-x'')\right\}-yx''+xy''=0$$

reducible to

$$y^2+x^2+\mathrm{A}y-\mathrm{B}x=0$$
$$y^2+x^2-\mathrm{C}y+\mathrm{D}x=0$$

Where

$$\mathrm{A}=x'\cot\alpha-y' \qquad \mathrm{B}=y'\cot\alpha+x'$$
$$\mathrm{C}=x''\cot\beta+y'' \qquad \mathrm{D}=y''\cot\beta-x''$$

Then

$$\frac{y}{x}=\frac{\mathrm{B}+\mathrm{D}}{\mathrm{A}+\mathrm{C}}=m$$

$$m^2x+x=\mathrm{B}-m\mathrm{A}$$

$$x=\frac{\mathrm{B}-m\mathrm{A}}{m^2+1} \qquad y=mx$$

Example.

A −1811·59	−1018·55	$y' = -1376·55$	$x' = +406·90$
B + 6·81	−930·26	$y'' = +441·85$	$x'' = +495·19$
C − 435·04	−1425·45	0·00	0·00

$\alpha = 64·7·40$

$\beta = 20·33·20$

+9·685719	+9·685719	+0·425980	+0·425980
+2·609488	−3·138792	+2·694772	+2·645275
+2·295207	−2·824511	+3·120752	+3·071255
+197·34	−667·59	+1320·54	+1178·30
$-y'$ +1376·55	$+x'$ +406·90	$+y''$ +441·85	$-x''$ −495·19
A +1573·89	B −260·69	C +1762·39	D +683·11
A +3·196974	−2·662730	A +1573·89	B −260·69
m 9·102482	0·006906	+3336·28	+422·42
+2·299456	x −2·655824	2·625744	m 9·102482
$-m$A −199·28	9 102482	3·523262	m^2 8·204964
+B −260·69	y −1·758306	m^2+1	=1·01603
−459·97		y −57·32	x −452·71
		−435·04	−1425·45
	Co-ordinates of P :	−492·36	−1878·16

A check is afforded by the computation of the angles of direction PA and PB. PC is given by

$$\tan^{-1} m.$$

March, 1898.